

SOME APPROXIMATION RESULTS ON A CLASS
OF NEW TYPE λ -BERNSTEIN POLYNOMIALS

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Abstract. The main concern of this article is to acquire some approximation properties of a new class of Bernstein polynomials based on Bézier basis functions with shape parameter $\lambda \in [-1, 1]$. We prove Korovkin type approximation theorem and estimate the degree of convergence in terms of the modulus of continuity, for the functions belong to Lipschitz type class and Peetre's K -functional, respectively. Additionally, with the help of Maple software, we present the comparison of the convergence of newly defined operators to the certain functions with some graphical illustrations and error estimation tables. Also, we conclude that the error estimation of our newly defined operators in some cases is better than classical Bernstein operators [3], Cai et al. [4] and Izgi [10].

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