

SOME GENERALIZED NUMERICAL RADIUS INEQUALITIES FOR HILBERT SPACE OPERATORS

M. H. M. RASHID* AND N. H. ALTAWEEL

Abstract. Some generalizations and refinements inequalities for the operator norm and numerical radius of the product and sum of Hilbert space operators are established. Refinements of some famous norm operators and numerical radius inequalities are also pointed out. As shown in this work, these refinements generalize and refine some recent and old results obtained in the literature.

Mathematics subject classification (2020): Primary 47A12, 47A30, 47B15; Secondary 47A63.

Keywords and phrases: Numerical radius, convex function operator, mixed Schwarz inequality, operator norm, Young inequality.

REFERENCES

- [1] J. S. AUJLA AND F. C. SILVA, *Weak majorization inequalities and convex functions*, Linear Algebra Appl., **369** (2003), 217–233.
- [2] S. S. DRAGOMIR, *Some refinements of schwarz inequality*, Simposional de Math Si Appl polytechnical Inst Timisoara, Romania, **1–2** (1985), 13–16.
- [3] S. S. DRAGOMIR, *Reverse inequalities for the numerical radius of linear operators in Hilbert spaces*, Bull. Austral. Math. Soc., **73** (2) (2006), 255–262.
- [4] S. S. DRAGOMIR, *Power inequalities for the numerical radius of a product of two operators in Hilbert spaces*, Seraj. J. math. **5** (18) (2009), 269–278.
- [5] S. S. DRAGOMIR, *Some inequalities generalizing Kato's and Furuta's results*, Filomat, **28** (1)(2014), 179–195.
- [6] K. GUSTAFSON AND D. RAO, *Numerical range*, Springer-Verlage, New York, 1997.
- [7] M. HAJMOHAMADI, R. LASHKARIPOUR AND M. BAKHERAD, *Further refinements of generalized numerical radius inequalities for Hilbert space operators*, Georgian Math. J., **28** (1) (2021), 83–92.
- [8] T. KATO, *Notes on some inequalities for linear operators*, Math. Ann., **125** (9) (1952), 208–212.
- [9] F. KITTANEH, *Notes on some inequalities for Hilbert Space operators*, Publ Res. Inst. Math. Sci., **24** (2) (1988), 283–293.
- [10] F. KITTANEH, *A numerical radius inequality and an estimate for the numerical radius of the Frobenius companion matrix*, Studia Math., **158** (1) (2003), 11–17.
- [11] F. KITTANEH, *Numerical radius inequalities for Hilbert space operators*, Studia Math., **168** (1) (2005), 73–80.
- [12] F. KITTANEH, *Spectral radius inequalities for Hilbert space operators*, Proc. Amer. Math. Soc., **134** (2) (2005), 385–390.
- [13] F. KITTANEH, *Norm inequalities for certain operator sums*, J. Funct. Anal., **143** (2) (1997), 337–348.
- [14] F. KITTANEH AND M. EL-HADDAD, *Numerical radius inequalities for Hilbert space operators II*, Studia Math., **182** (2) (2007), 133–140.
- [15] Y. AL-MANASRAH AND F. KITTANEH, *A generalization of two refined Young inequalities*, Positivity, **19** (2015), 757–768.
- [16] H. NAJAFI, *Some numerical radius inequality for several operators*, Linear Algebra Appl., **588** (2020), 489–496.

- [17] J. PEMCARIŠĆ, T. FURUTA, J. MIŠČIŠĆ HOT, AND Y. SEO, *Mondpencarić method in operator inequalities, Inequalities for Bounded Selfadjoint Operators on a Hilbert Space*, Element, Zagreb, 2005.
- [18] M. H. M. RASHID, *Power inequalities for the numerical radius of operators in Hilbert spaces*, Khayyam J. Math., **5** (2) (2019), 15–29.
- [19] M. H. M. RASHID AND N. H. ALTAWEEL, *Numerical range of generalized aluthge transformation*, Informatica J. **32** (5) (2021), 2–11.
- [20] TH. M. RASSIAS, *Survey on Classical Inequalities*, Kluwer Academic Publishers, Dordrecht, 2000.
- [21] M. SATTARI, M. S. MOSLEHIAN AND T. YAMAZAKI, *Some generalized numerical radius inequalities for Hilbert space operators*, Linear Algebra Appl. **470** (2015), 216–227
- [22] A. ZAMANI, M. MOSLEHIAN, Q. XU AND C. FU, *Numerical radius inequalities concerning with algebra norms*, Mediterr. J. Math., **18** (2) (2021), 38–31.