

DEVIATION ESTIMATIONS FOR LOTKA–NAGAEV ESTIMATOR OF A BRANCHING PROCESS WITH IMMIGRATION

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Abstract. In a recent manuscript, Chu (2018) applied the self-normalized large deviations for i.i.d. random variables to the Lotka–Nagaev estimation of a supercritical Galton–Watson process. In this paper, we consider decay rates for the Lotka–Nagaev estimation of a supercritical branching process with immigration. We have two main contributions. On the one hand, Chu’s paper considered the self-normalizing constants of second order, otherwise, we consider the maximum case. On the other hand, except for large deviations, we also studied the self-normalized moderate deviations. The classical large deviation probabilities for Lotka–Nagaev estimation show three different decay rates according to the degree of heavy tail of offspring distribution, but our results show that there is only one decay rate in the self-normalized version.

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REFERENCES

- [1] K. B. ATHREYA, *Large deviation rates for branching processes. I: single type case*, Ann. Appl. Probab., **4**, 3 (1994), 779–790.
- [2] W. J. CHU, *Self-normalized large deviation for supercritical branching processes*, J. Appl. Prob., **52**, 2 (2018), 450–458.
- [3] A. DEMBO AND O. ZEITOUNI, *Large deviations techniques and applications*, New York: Springer-Verlag.
- [4] K. FLEISCHMANN AND V. WACHTEL, *Large deviations for sums indexed by the generations of a Galton–Watson process*, Probab. Theory Relat. Fields, **141**, 2 (2008), 455–470.
- [5] H. HE, *On large deviation rates for sums associated with Galton–Watson processes*, Adv. Appl. Probab., **48**, 3 (2016), 672–690.
- [6] C. C. HEYDE AND E. SENETA, *Analogues of classical limit theorems for the supercritical Galton–Watson process with immigration*, Mathematical Biosciences, **3–4**, (1971), 249–259.
- [7] L. Y. LI, J. P. LI, *Large deviation rates for supercritical branching processes with immigration*, Journal of Theoretical Probability, **34**, (2021), 162–172.
- [8] J. N. LIU AND M. ZHANG, *Large deviation for supercritical branching processes with immigration*, Acta Mathematica Sinica, **32**, 8 (2016), 893–900.
- [9] P. E. NEY AND A. N. VIDYASHANKAR, *Harmonic moments and large deviation rates for supercritical branching processes*, Ann. Appl. Probab., **13**, 2 (2003), 475–489.
- [10] L. V. ROZOVSKY, *The chernoff-type large deviations for sums of a random normalization*, Theory Probab. Appl., **53**, 4 (2009), 727–735.
- [11] Q. M. SHAO, *Self-normalized Large deviations*, Ann. Probab., **25**, 1 (1997), 285–328.
- [12] Q. SUN AND M. ZHANG, *Harmonic moments and large deviations for supercritical branching processes with immigration*, Front. Math. China, **12**, 5 (2017), 1201–1220.