

## THRESHOLD DYNAMICS BEHAVIORS OF A STOCHASTIC SIRS EPIDEMIC MODEL WITH A PARAMETER FUNCTIONAL VALUE

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**Abstract.** This article pays the main attention to the notions of a spreading threshold dynamical model for a stochastic SIRS with environmental noise. A unique positive solution of the stochastic model is proved to be existed in this article. Furthermore, by appropriate Lyapunov functions, the ergodic stationary distribution is introduced. The conditions of extinction or permanence of the SIRS epidemic model are also considered in this article.

**Mathematics subject classification (2020):** 37A50, 37H05, 37N25.

**Keywords and phrases:** Stochastic SIRS model, Lyapunov function, ergodicity, extinction, environmental noise.

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