

## CONSTRUCTION OF THE KANTOROVICH VARIANT OF THE BERNSTEIN–CHLODOVSKY OPERATORS BASED ON PARAMETER $\alpha$

BO-YONG LIAN AND QING-BO CAI\*

**Abstract.** In this article, a new family of Kantorovich variant of Chlodovsky operators is introduced. The authors establish some approximation theorems, such as a direct approximation by means of the Ditzian-Totik modulus of smoothness, a global approximation theorem in terms of second order modulus of continuity and so on. Furthermore, a Voronovskaja type asymptotic estimate formula is presented. Finally, the rate of convergence for some absolutely continuous functions having a derivative equivalent to a bounded variation function is obtained.

**Mathematics subject classification (2020):** 41A36, 41A25, 41A10.

**Keywords and phrases:** Chlodovsky operators, modulus of smoothness, modulus of continuity, bounded variation.

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