

## WEIGHTED A-STATISTICAL CONVERGENCE AND BÖGEL APPROXIMATION BY OPERATORS OF EXPONENTIAL TYPE

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**Abstract.** We give a proper definition of an exponential type operator proposed by Ismail and May [21] so that it acts from  $C(S)$  into  $C(S)$  and explore its non-multiplicativity,  $r^{\text{th}}$  order generalization and weighted A-statistical convergence in the univariate case. Next, we define properly the associated tensor product of the operators and investigate its approximation properties. Lastly, we introduce the associated Generalized Boolean Sum (GBS) operators and present error estimates using mixed modulus of smoothness for Bögel continuous functions.

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