

MONOTONIC PROPERTIES FOR RATIO OF THE GENERALIZED (p, k) -POLYGAMMA FUNCTIONS

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Abstract. In this paper, we show monotonic properties for ratio of the generalized (p, k) -polygamma functions by using Mehrez-Sitnik method. The new inequalities extend the known result obtained by Feng Qi.

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