

## RANDOM STABILITY AND HYPERSTABILITY OF MULTI-QUADRATIC MAPPINGS

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*Abstract.* In this paper, we introduce a new quadratic functional equation. In light of this equation, we define the multi-quadratic mappings and reduce the system of  $n$  equations defining the multi-quadratic mappings to a single equation. We also obtain some stability and hyperstability results concerning multi-quadratic mappings in the setting of random normed spaces.

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