

DEFERRED STATISTICAL CONVERGENCE AND POWER SUMMABILITY METHOD FOR q -LAGUERRE POLYNOMIALS OPERATOR

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Abstract. In the present article, we discuss the Korovkin type approximation theorems and the rate of convergence with the aid of the modulus of continuity using deferred statistical convergence and the power series summability technique for an operator based on q -Laguerre polynomials introduced by Özarşlan (Studia Sci. Math. Hungar., 44 (1), 65–80). We also define the r -th order generalization of these operators by means of the Taylor polynomial to approximate functions in $f \in C^r[0, 1]$ such that $f^{(r)} \in Lip_K \alpha$, $0 < \alpha \leq 1$. Furthermore, we find an estimate of the rate of convergence of the q -Laguerre operator acting on f at those points x where the one sided q -derivatives $\mathfrak{D}_q^+ f$ and $\mathfrak{D}_q^- f$ exist.

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