

A MORE ACCURATE EXTENDED HARDY–HILBERT'S INEQUALITY WITH PARAMETERS

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Abstract. In this paper, in virtue of the symmetry principle, applying the techniques of real analysis and Euler-Maclaurin summation formula, we construct proper weight coefficients and use them to establish a more accurate extended Hardy-Hilbert's inequality with parameters. Then, we obtain the equivalent forms and some equivalent statements of the best possible constant factor related to several parameters. Finally, we illustrate the operator expressions and how the obtained results can generate some particular Hardy-Hilbert's inequalities.

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