

SOME PARAMETERIZED INEQUALITIES ARISING FROM THE TEMPERED FRACTIONAL INTEGRALS INVOLVING THE (μ, η) -INCOMPLETE GAMMA FUNCTIONS

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Abstract. We utilize the definition of a class of the tempered fractional integral operators, proposed by Sabzikar et al. in [J. Comput. Phys., 293:14–28, 2015], to establish a fractional-type integral identity with parameters. We present some parameterized integral inequalities for differentiable mappings in terms of the constructed identity, and give two examples to identify the correctness of the obtained results as well.

Mathematics subject classification (2020): 26A33, 41A55, 26D15, 26E60.

Keywords and phrases: Hermite–Hadamard-type inequalities, (μ, η) -incomplete gamma function, fractional integrals.

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