

ON BOUNDEDNESS AND COMPACTNESS OF DISCRETE HARDY OPERATOR IN DISCRETE WEIGHTED VARIABLE LEBESGUE SPACES

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Abstract. In this paper, the authors establish a two-weight boundedness criterion of discrete Hardy operator and its dual operator in the scale of discrete weighted variable Lebesgue spaces. Moreover, we study the problem of compactness of the discrete Hardy operator in discrete weighted variable Lebesgue spaces. We also study a similar problem for the dual operator of discrete Hardy operator.

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REFERENCES

- [1] K. F. ANDERSEN AND H. P. HEINIG, *Weighted norm inequalities for certain integral operators*, SIAM J. Math., 14 (4) (1983), 834–844.
- [2] R. A. BANDALIEV, *The boundedness of certain sublinear operator in the weighted variable Lebesgue spaces*, Czechoslovak Math. J., 60 (2) (2010), 327–337, corrigendum in Czechoslovak Math. J., 63 (4) (2013), 1149–1152.
- [3] R. A. BANDALIEV, *The boundedness of multidimensional Hardy operator in the weighted variable Lebesgue spaces*, Lith. Math. J., 50 (3) (2010), 249–259.
- [4] R. A. BANDALIEV, *On Hardy-type inequalities in weighted variable Lebesgue space $L_{p(x)}$ for $0 < p(x) < 1$* , Eurasian Math. J., 4 (4) (2013), 5–16.
- [5] R. A. BANDALIYEV, A. SERBETCI AND S. G. HASANOV, *On Hardy inequality in variable Lebesgue spaces with mixed norm*, Indian J. Pure Appl. Math., 49 (2018), 765–782.
- [6] G. BENNETT, *Some elementary inequalities*, Quart. J. Math. Oxford Ser. 2, 38 (4) (1987), 401–425.
- [7] G. BENNETT, *Some elementary inequalities II*, Quart. J. Math. Oxford Ser. 2, 39 (4) (1988), 385–400.
- [8] G. BENNETT, *Some elementary inequalities III*, Quart. J. Math. Oxford Ser. 2, 42 (1) (1991), 149–174.
- [9] M. S. BRAVERMAN AND V. D. STEPANOV, *On the discrete Hardy inequality*, Bull. London Math. Soc., 26 (3) (1994), 283–287.
- [10] D. CRUZ-URIBE AND A. FIORENZA, *Variable Lebesgue spaces: Foundations and harmonic analysis*, Birkhauser, Basel (2013).
- [11] D. CRUZ-URIBE AND F. I. MAMEDOV, *On a general weighted Hardy type inequality in the variable exponent Lebesgue spaces*, Rev. Mat. Complut., 25 (2) (2012), 335–367.
- [12] L. DIENING, P. HARJULEHTO, P. HÄSTÖ AND M. RUŽIČKA, *Lebesgue and Sobolev spaces with variable exponents*, Lecture Notes in Math., 2017, Springer, Berlin (2011).
- [13] L. DIENING AND S. SAMKO, *Hardy inequality in variable exponent Lebesgue spaces*, Frac. Calc. Appl. Anal., 10 (1) (2007), 1–18.
- [14] A. GOGATISHVILI AND V. D. STEPANOV, *Reduction theorems for weighted integral inequalities on the cone of monotone functions*, Russian Math. Surveys, 68 (4) (2013), 597–664.
- [15] M. L. GOL'DMAN, *Sharp estimates of the norms of Hardy-type operators on the cone of quasimonotone functions*, Proc. Steklov Inst. Math., 232 (2001), 109–137.
- [16] P. GÓRKA AND A. MACIOS, *Almost everything you need to know about relatively compact sets in variable Lebesgue spaces*, J. Funct. Anal., 269 (7) (2015), 1925–1949.

- [17] G. H. HARDY, J. E. LITTLEWOOD AND G. POLYA, *Inequalities*, Cambridge University Press, Cambridge (1934).
- [18] H. P. HEINIG, *Weighted inequalities for certain integral operators II*, Proc. Amer. Math. Soc., 95 (3) (1985), 387–395.
- [19] A. A. KALYBAY, R. OINAROV AND A. M. TEMIRKHANOVA, *Boundedness of n -multiple discrete Hardy operators with weights for $1 < q < p < \infty$* , J. Funct. Spaces Appl. 2013, Art. ID 121767(2013), 1–9.
- [20] A. A. KALYBAY, L.-E. PERSSON AND A. M. TEMIRKHANOVA, *A new discrete Hardy-type inequality with kernels and monotone functions*, J. Inequal. Appl. 2015, 2015:321, 1–10.
- [21] T. S. KOPALIANI, *On some structural properties of Banach function spaces and boundedness of certain integral operators*, Czechoslovak Math. J. 54 (3) (2004), 791–805.
- [22] A. KUFNER, L. MALIGRANDA AND L.-E. PERSSON, *The prehistory of the Hardy inequality*, Amer. Math. Monthly, 113 (8) (2006), 715–732.
- [23] F. I. MAMEDOV AND A. HARMAN, *On a weighted inequality of Hardy type in spaces $L^{p(\cdot)}$* , J. Math. Anal. Appl., 353 (2) (2009), 521–530.
- [24] F. I. MAMEDOV, F. MAMMADOVA AND M. ALIYEV, *Boundedness criterions for the Hardy operator in weighted $L^{p(\cdot)}(0, l)$ space*, J. Convex Anal., 22 (2) (2015), 553–568.
- [25] J. MUSIELAK, *Orlicz spaces and modular spaces*, Lecture Notes in Math., 1034, Springer, Berlin (1983).
- [26] J. MUSIELAK AND W. ORLICZ, *On modular spaces*, Stud. Math., 18 (1959), 49–65.
- [27] H. NAKANO, *Modulare semi-ordered linear spaces*, Maruzen, Co., Ltd., Tokyo (1950).
- [28] H. NAKANO, *Topology and topological linear spaces*, Maruzen, Co., Ltd., Tokyo (1951).
- [29] A. NEKVINDA, *Embeddings between discrete weighted Lebesgue spaces with variable exponents*, Math. Inequal. Appl., 10 (1) (2007), 165–172.
- [30] C. A. OKPOTI, L.-E. PERSSON AND A. WEDESTIG, *Scales of weight characterizations for the discrete Hardy and Carleman inequalities*, Proceedings of Function Spaces, Differential Operators and Nonlinear Analysis (FSDONA' 04), Acad. Sci. Czech Rep., Milovy, 2004, 236–258.
- [31] C. A. OKPOTI, L.-E. PERSSON AND A. WEDESTIG, *Weight characterizations for the discrete Hardy inequality with kernel*, J. Inequal. Appl., 2006, 18030 (2006), 1–14.
- [32] W. ORLICZ, *Über konjugierte exponentenfolgen*, Stud. Math., 3 (1931), 200–212.