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A DERIVATIVE–HILBERT OPERATOR ACTING FROM BESOV SPACES INTO BLOCH SPACE

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Abstract. If μ is a positive Borel measure on the interval $[0,1)$, we let $\mathscr{H}_\mu$ be the Hankel matrix $\mathscr{H}_\mu = (\mu_{n,k})_{n,k\geqslant 0}$ with entries $\mu_{n,k} = \mu_{n+k}$ and $\mu_n = \int_{[0,1)} t^n d\mu(t)$. Using $\mathscr{H}_\mu$, Ye and Zhou first defined the Derivative-Hilbert operator as

$$\mathscr{D}\mathscr{H}_\mu(f)(z) = \sum_{n=0}^{\infty}\left(\sum_{k=0}^{\infty}\mu_{n,k}a_k\right)(n+1)z^n, \quad z\in\mathbb{D},$$

where $f(z) = \sum_{n=0}^{\infty} a_n z^n$ is an analytic function in $\mathbb{D}$. In this paper, we characterize the measure μ for which $\mathscr{D}\mathscr{H}_\mu$ is a bounded (resp., compact) operator from Besov space B_p into Bloch space $\mathscr{B}$ with $1 < p < \infty$.

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Paper JMI-16-82

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