

COMPLETE CONVERGENCE AND COMPLETE MOMENT CONVERGENCE FOR ARRAYS OF ROWWISE NEGATIVELY DEPENDENT RANDOM VARIABLES UNDER SUB-LINEAR EXPECTATIONS

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Abstract. In this paper, under some suitable conditions, we study the complete convergence and complete moment convergence for arrays of rowwise negatively dependent random variables in sub-linear expectation space $(\Omega, \mathcal{H}, \mathbb{E})$. Some general results on complete convergence and complete moment convergence for arrays of rowwise negatively dependent random variables under sub-linear expectations are established, which extend the corresponding ones in classical probability space to the case of sub-linear expectation space.

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