CATER TYPE INEQUALITIES INVOLVING CATER
PRODUCTS AND THEIR APPLICATIONS IN SPACE SCIENCE

JIA JIN WEN, TIAN YONG HAN AND JUN YUAN*

Abstract. By means of the mathematical induction, stepwise adjustment method and the reorder method, under the proper hypotheses, we established the following Cater type inequalities involving Cater products:

$$X \otimes Y \geqslant KX \otimes Y \geqslant K^+ X \otimes Y > e^{-1} \text{ and } f \otimes g \geqslant f(1-t) \otimes g > e^{-1}.$$

As applications, we solved the problem which proposed by M. Laub, Jerusalem and Israel in under the proper hypotheses, and an $\mathbf{l}$-isoperimetric inequality in the centered n-surround system $S^{(2)}\{P, \Gamma, \mathbf{l}\}$ is obtained as follows:

$$[\mu] \otimes [\mathbf{l}] \geqslant \left(\frac{|\Gamma|}{n} \right)^{\frac{2\pi}{n}}.$$

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REFERENCES

[1] *Problems and Solutions*, Amer. Math. Monthly, **87** (4) (1980), 302–303.
[2] PROBLEMS AND SOLUTIONS, Amer. Math. Monthly, **97** (1) (1990), 65–67.
[3] A. M. ACU AND M. D. RUSU, *New results concerning Chebyshev-Grüss-type inequalities via discrete oscillations*, Appl. Math. Comput., **243** (2014), 585–593.
[4] H. AGAHI, R. MESIAR AND Y. OUYANG, *Chebyshev type inequalities for pseudo-integrals*, Nonlinear Anal., Theory Methods and Applications, **72** (6) (2010), 2737–2743.
[5] G. ANASTASSIOU, *Chebyshev-Grüss type inequalities on* $\mathbb{R}^N$ *over spherical shells and balls*, Appl. Math. Lett., **21** (2008), 119–127.
[6] A. COLESANTI, M. LUDWIG AND F. MUSSNIG, *A homogeneous decomposition theorem for valuations on convex functions*, J. Funct. Anal., **279** (5) (2020), Paper No. 108573, 25 pp.
[7] R. CORREA, A. HANTOUTE AND P. PĚREZ-AROS, *Characterizations of the subdifferential of convex integral functions under qualification conditions*, J. Funct. Anal., **277** (1) (2019), 227–254.
[8] N. DAFNIS, A. GIANNOPOULOS AND A. TSOLOMITIS, *Asymptotic shape of a random polytope in a convex body*, J. Funct. Anal. 257 (2009), no. 9, 2820–2839.
[9] B. DARABY, *Investigation of a Stolarsky type inequality for integrals in pseudo-analysis*, Fract. Calc. Appl. Anal., **13** (5) (2010), 467–473.
[10] J. DASCĂL AND J. JARCZYK, *Computer assisted solution of an equality problem of mean values*, Appl. Math. Comput., **219** (2) (2012), 475–481.
[11] K. E. DIETHELM, *The mean value theorems and a Nagumo-type uniqueness theorem for Caputo's fractional calculus*, Fract. Calc. Appl. Anal., **20** (6) (2017), 1567–1570.
[12] J. DOU AND M. ZHU, L^1 *isoperimetric inequality on the unit disk*, Adv. Math., **404** (2022), Paper No. 108383, 25 pp.
[13] C. B. GAO AND J. J. WEN, *A dimensionality reduction principle on the optimization of function*, J. Math. Inequal., **7** (3) (2013), 357–375.

[14] C. B. GAO AND J. J. WEN, *Theories and inequalities on the satellite system*, ISRN Math. Anal., **2011**, Article ID 909261, 22 pages.

[15] C. B. GAO AND J. J. WEN, *Theory of surround system and associated inequalities*, Comput. Math. Appl., **63** (2012), 1621–1640.

[16] D. Y. HWANG AND S. S. DRAGOMIR, *Comparing two integral means for absolutely continuous functions whose absolute value of the derivative are convex and applications*, Appl. Math. Comput., **230** (2014), 259–266.

[17] K. JOTSAROOP AND S. SHRIVASTAVA, *Maximal estimates for bilinear Bochner-Riesz means*, Adv. Math., **395** (2022), Paper No. 108100, 38 pp.

[18] Y. M. JUNG, J. J. WHANG AND S. YUN, *Sparse probabilistic K-means*, Appl. Math. Comput., **382** (2020), 125328, 12 pp.

[19] J. KNOERR, *The support of dually epi-translation invariant valuations on convex functions*, J. Funct. Anal., **281** (5) (2021), Paper No. 109059, 52 pp.

[20] L. M. MEAUX, J. W. J. SEAMAN AND T. L. BOULLION, *Calculation of multivariate Chebyshev-type inequalities*, Comput. Math. Appl., **20** (12) (1990), 55–60.

[21] Y. OUYANG, R. MESIAR AND J. LI, *On the comonotonic-⋆-property for Sugeno integral*, Appl. Math. Comput., **211** (2) (2009), 450–458.

[22] J. E. PEČARIĆ, J. J. WEN, W. L. WANG AND T. LU, *A generalization of Maclaurin's inequalities and its applications*, Math. Inequal. Appl., **8** (4) (2005), 583–598.

[23] S. PONNUSAMY, J. QIAO AND X. WANG, *Schwarzian derivative, integral means, and the affine and linear invariant families of biharmonic mappings*, Appl. Math. Comput., **216** (12) (2010), 3468–3479.

[24] G. UR RAHMAN, R. P. AGARWAL AND Q. DIN, *Mathematical analysis of giving up smoking model via harmonic mean type incidence rate*, Appl. Math. Comput., **354** (2019), 128–148.

[25] A. WAGENE, *Chebyshevs algebraic inequality and comparative statics under uncertainty*, Math. Soc. Sci., **52** (2006), 217–221.

[26] J. J. WEN AND C. B. GAO, *Geometric inequalities involving the central distance of the centered 2-surround system*, Acta. Math. Sinica. **51** (4) (2008), 815–832, (in Chinese).

[27] J. J. WEN, C. B. GAO AND W. L. WANG, *Inequalities of J-P-S-F type*, J. Math. Inequal., **7** (2) (2013), 213–225.

[28] J. J. WEN, T. Y. HAN AND J. YUAN, *Stability inequalities involving gravity norm and temperature*, J. Math. Inequal., **14** (4) (2020), 1007–1037.

[29] J. J. WEN, J. E. PEČARIĆ AND T. Y. HAN, *Weak monotonicity and Chebyshev type inequality*, Math. Inequal. Appl., **18** (1) (2015), 217–231.

[30] J. J. WEN AND W. L. WANG, *Chebyshev type inequalities involving permanents and their applications*, Linear Algebra Appl., **422** (1) (2007), 295–303.

[31] J. J. WEN AND W. L. WANG, *The optimization for the inequalities of power means*, J. Inequal. Appl., **2006**, Article ID 46782, 25 pages.

[32] J. J. WEN, S. H. WU, J. YUAN AND T. Y. HAN, *Mean central distance–central distance inequalities*, J. Math. Inequal., **11** (4) (2017), 1131–1149.

[33] J. J. WEN, J. YUAN AND S. H. WU, *Isoperimetric inequalities in surround system and space science*, J. Inequal. Appl., **2016**: 74 (2016), 28 pp.

[34] J. J. WEN, J. YUAN, S. H. WU AND T. Y. HAN, *Gravity inequalities and the mean temperature on a planet*, J. Inequal. Appl., **2016**: 264, 18 pp.

[35] YEWALE, R. BHAGWAT, PACHPATTE AND B. DEEPAK, *Some new Chebyshev type inequalities via extended generalized fractional integral operator*, J. Fract. Calc. Appl., **12** (2) (2021), 11–19.