

NUMERICAL RANGES OF SUM OF TWO WEIGHTED COMPOSITION OPERATORS ON THE HARDY SPACE H^2

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Abstract. Let φ be an analytic self-map of the open unit disk \mathbb{D} and let ψ be an analytic function on \mathbb{D} . The weighted composition operator $C_{\psi, \varphi}$ is the operator on the Hardy space H^2 given by $C_{\psi, \varphi} f = \psi f \circ \varphi$. Under some conditions on φ_1 and φ_2 , we try to find a subset of the numerical range of $C_{\psi_1, \varphi_1} + C_{\psi_2, \varphi_2}$ and determine when zero lies in the interior of the numerical range of $C_{\psi_1, \varphi_1} + C_{\psi_2, \varphi_2}$.

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