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NEW WEIGHTED HERMITE–HADAMARD TYPE INEQUALITIES FOR DIFFERENTIABLE STRONGLY CONVEX AND STRONGLY QUASI–CONVEX MAPPINGS

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Abstract. In this paper, new weighted Hermite-Hadamard type inequalities for differentiable strongly convex and strongly quasi-convex mappings are proved. These results strengthen many results proved in earlier works for these classes of functions. Applications of some of our results to statistics are provided.

Mathematics subject classification (2020): 26D15, 26D20, 26D07.
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