

## HARNACK INEQUALITIES FOR FUNCTIONAL SDES DRIVEN BY SUBORDINATE FRACTIONAL BROWNIAN MOTION

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*Abstract.* Being base on coupling by change of measure and an approximation technique, the Harnack inequalities for a class of stochastic functional differential equations driven by subordinate fractional Brownian motion with Hurst parameter  $0 < H < 1/2$  are established. By using a transformation formulas for fractional Brownian motion, the Harnack inequalities for stochastic functional differential equations driven by subordinate fractional Brownian motion with Hurst parameter  $1/2 < H < 1$  are established.

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