

COMBINATORIAL INEQUALITIES ARISING FROM THE INCLUSION–EXCLUSION PRINCIPLE

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Abstract. We study a new inequality arising from the principle of inclusion and exclusion by mixing the idea from Hlawka's inequality and Tverberg's combinatorial sum. We obtain sharp lower bounds for the sum when the number of variables is small.

Mathematics subject classification (2020): Primary 26D15; Secondary 05A20, 26D07.

Keywords and phrases: Hlawka's inequality, Popoviciu's inequality, triangle inequality, inclusion-exclusion principle.

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