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THE SHARPER VERSION FOR GENERALIZED POWER MEAN INEQUALITIES WITH NEGATIVE EXPONENT

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Abstract. In this study, the generalized power mean inequalities with a negative parameter are refined using an optimality theorem on the generator function. The optimality theorem requires the study of different cases for the exponents and yields a refinement of the inequality in a neighbourhood of the vectors for which the equality occurs. Then, these local inequalities are generalized to all positive vectors by an appropriate selection of parameters. Also, some of the results are exemplified by numerical calculations.

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