

## ALGEBRAIC INTEGERS WITH SMALL POTENTIAL ENERGY

ARTŪRAS DUBICKAS

*Abstract.* In this paper we give a lower bound on the mean of squares of distances between the points in two sets in terms of the products of distances between the points in each of those two sets. These results imply lower bounds on the  $p$ th power ( $p \geq 2$ ) of the average distance between the conjugates of an algebraic integer and of the same quantity between the conjugates of a totally real algebraic integer.

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