

ON ARITHMETIC–GEOMETRIC AND GEOMETRIC–ARITHMETIC INDICES OF GRAPHS

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Abstract. Let G be a connected graph having vertex set $\{v_1, \dots, v_n\}$ and vertex-degree sequence (d_1, \dots, d_n) , where d_i represents the degree of the vertex v_i . If the vertices v_i and v_j are adjacent in G , we write $i \sim j$. The arithmetic–geometric index and the geometric–arithmetic index of G are defined as $AG(G) = \sum_{i \sim j} [(d_i + d_j) / (2\sqrt{d_i d_j})]$ and $GA(G) = \sum_{i \sim j} [2\sqrt{d_i d_j} / (d_i + d_j)]$, respectively. Since $AG(G)$ and $GA(G)$ are closely related quantities, we derive bounds on their addition as well as on their difference, namely on $irr_{AG}(G) = AG(G) - GA(G)$ and $r(G) = AG(G) + GA(G)$. Some new bounds on $AG(G)$ are also obtained.

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