

A NEW HILBERT-TYPE INTEGRAL INEQUALITY WITH THE GENERAL NONHOMOGENEOUS KERNEL AND APPLICATIONS

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Abstract. In this paper, by the use of weight functions and the technique of real analysis, a new Hilbert-type integral inequality with a general nonhomogeneous kernel as $H(x\nu(y))$ is given. A few equivalent statements related to the best possible constant factor and parameters are considered. As applications, the equivalent forms, the operator expressions and some corollaries are obtained.

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