SOME INEQUALITIES ON THE CONVERGENT
ABSCISSAS OF LAPLACE–STIELTJES TRANSFORMS

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Abstract. By making use of the basic properties of Laplace-Stieltjes transform, we establish some inequalities concerning the abscissa of convergence, the abscissa of absolute convergence and the abscissa of uniform convergence of Laplace-Stieltjes transform $\int_0^\infty e^{st} d\alpha(t)$. Moreover, we point out that our formulas for the three abscissas of convergence are consistent with the previous results given by Yu [26] for Laplace-Stieltjes transform under some conditions.

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