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ON THE LOCAL RITT RESOLVENT CONDITION

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Abstract. Let T be a linear bounded operator on a complex Banach space $\mathscr{X}$. In this paper, we introduce a local version of the Ritt resolvent condition $[LR]$ for Banach space operators T. We start by showing that this concept is weaker than the classical Ritt condition $[R]$. We prove that, for operators with single-valued extension property (SVEP), estimate $[LR]$ extends, with a larger constant, to some sector K_δ. Moreover, by extending some Ritt's theorems to the local case for operators with the SVEP, several characterizations of the local sublinear decay of $T^n - T^{n+1}$ have been established.

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