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SOME NEW NUMERICAL RADIUS AND HILBERT–SCHMIDT NUMERICAL RADIUS INEQUALITIES FOR HILBERT SPACE OPERATORS

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Abstract. In this article, we give new upper and lower bounds of numerical radius and Hilbert-Schmidt numerical radius inequalities for Hilbert space operators. In particular, we show that if $X \in C_2$ with the Cartesian decomposition $X = A + iB$, then

$$\frac{1}{4}\||X|^2 + |X^*|^2\|_2 \leqslant \frac{1}{\sqrt{2}}\omega_2\left(\begin{bmatrix} 0 & A^2 \\ B^2 & 0 \end{bmatrix}\right) \leqslant \omega_2^2(X).$$

This is an analog of Kittaneh in [Studia Math. 168 (2005): 73-80].

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