CERTAIN FRACTIONAL INTEGRAL INCLUSIONS PERTAINING TO INTERVAL–VALUED EXPONENTIAL TRIGONOMETRIC CONVEX FUNCTIONS

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Abstract. As an interesting generalization involving the interval-valued convex functions, the interval-valued exponential trigonometric convex function is firstly introduced, and their meaningful properties are then investigated. Meanwhile, certain Hermite–Hadamard- and Pachpatte-type integral inclusion relations are also developed via the newly proposed functions in interval-valued fractional calculus. In particular, an improved version of the Hermite–Hadamard's integral inclusions pertaining to the interval-valued exponential trigonometric convex functions is proposed as well. To identify the correctness of the derived inclusion relations in the study, the graphical representations for the outcomes are provided in terms of the change of the parameter α.

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