

NON-UNIFORM BERRY-ESSEEN-TYPE INEQUALITIES FOR A SUPERCRITICAL BRANCHING PROCESS WITH IMMIGRATION IN A RANDOM ENVIRONMENT

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Abstract. Let W_n be the fundamental submartingale of a supercritical branching process with immigration in a random environment. In order to characterize the convergence rates of W_n , the quenched and annealed non-uniform Berry-Esseen-type inequalities are established for $W_{n+k} - W_n$ for each fixed $k \in \{1, 2, \dots, \infty\}$, which reveal the convergence rates of the corresponding central limit theorems.

Mathematics subject classification (2020): 60J80, 60K37, 60F05.

Keywords and phrases: Branching process with immigration, random environment, central limit theorem, Berry-Esseen inequality.

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