

EQUIVALENCE ON HIROSHIMA'S TYPE INEQUALITIES FOR POSITIVE SEMIDEFINITE BLOCK MATRICES

YUN ZHANG*, HAIBO ZHANG AND SHUO SHI

Abstract. In this paper, we prove that some Hiroshima's type inequalities for positive semidefinite block matrices are equivalent. These interesting results are due to Hiroshima [Phys. Rev. Lett. **91**, (2003), 057902], Lin and Wolkowicz [Linear Multilinear Algebra **60**, 11–12 (2012), 1365–1368], Turkmen, Paksoy and Zhang [Linear Algebra Appl. **437**, 6 (2012), 1305–1316], Zhang and Xu [J. Math. Inequal. **14**, 4 (2020), 1383–1388], respectively.

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