

INEQUALITIES FOR COMMUTATORS OF FRACTIONAL INTEGRALS AND SINGULAR INTEGRALS ON DIFFERENTIAL FORMS

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Abstract. In this paper, we define the commutators of fractional integral operators and Calderón-Zygmund singular integral operators on differential forms, and give the sufficient and necessary conditions for these commutators to be bounded on weighted Lebesgue spaces. As an application, the Caccioppoli-type inequalities with Orlicz norm for commutators of Calderón-Zygmund singular integral operators on differential forms are obtained.

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