

ASYMPTOTIC DISTRIBUTION OF THE WAVELET-BASED ESTIMATORS OF MULTIVARIATE REGRESSION FUNCTIONS UNDER WEAK DEPENDENCE

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Abstract. This paper investigates the nonparametric linear wavelet-based estimators of multivariate regression functions. Under mild conditions, we establish the asymptotic normality under the weak dependence, which incorporates mixing and association concepts. This framework applies to numerous classes of intriguing statistical processes, primarily Gaussian sequences and, more generally, Bernoulli shifts. We give an application for the confidence interval.

Mathematics subject classification (2020): 62E20, 60F05, 62G08, 62H12, 62G09.

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