

BLOCK DECOMPOSITION FOR HERZ SPACES ASSOCIATED WITH BALL BANACH FUNCTION SPACES

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Abstract. In this paper, we establish the block decomposition for Herz spaces associated with ball Banach function spaces. By using this decomposition, we obtain the boundedness for a class of sublinear operators on Herz spaces associated with ball Banach function spaces.

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