

ON SINGULAR INTEGRALS AND MAXIMAL OPERATORS ALONG SURFACES OF REVOLUTION ON PRODUCT DOMAINS

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Abstract. We study the mapping properties of singular integral operators along surfaces of revolutions on product domains. For several classes of surfaces, we prove sharp L^p bounds ($1 < p < \infty$) for these singular integral operators as well as their corresponding maximal operators. By using these L^p bounds and an extrapolation argument we obtain the L^p boundedness of these operators under optimal conditions on the singular kernels. Our results extend and improve several results previously obtained by many authors.

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REFERENCES

- [1] H. AL-QASSEM, *Singular integrals along surfaces on product domains*, Analysis in Theory and Applications, **20**, pp. 99–112 (2004).
- [2] H. AL-QASSEM AND A. AL-SALMAN, *L^p boundedness of a class of singular integral operators with rough kernels*, Turk. J. Math, **25** (2001), 519–533.
- [3] H. AL-QASSEM AND A. AL-SALMAN, *Rough singular integrals on product spaces*, International Journal of Mathematics and Mathematical Sciences, **67** (2004), 3671–3684.
- [4] H. AL-QASSEM, L. CHENG, AND Y. PAN, *On the boundedness of a class of rough maximal operators on product spaces*, Hokkaido Math. J., Vol. **40** (2011), 1–32.
- [5] H. AL-QASSEM AND Y. PAN, *L^p estimates for singular integrals with kernels belonging to certain block spaces*, Revista Matemática Iberoamericana (**18**) 3 (2002), 701–730.
- [6] H. AL-QASSEM AND Y. PAN, *L^p boundedness for singular integrals with rough kernels on product domains*, Hokkaido Math. J., **31** (2002), 555–613.
- [7] H. AL-QASSEM, Y. PAN, *Singular integrals along surfaces of revolution with rough kernels*, SUT J. Math. Vol. **39**, no. 1, (2003), 55–70.
- [8] H. AL-QASSEM AND Y. PAN, *On certain estimates for Marcinkiewicz integrals and extrapolation*, Collect. Math. **60**, 2 (2009), 123–145.
- [9] AL-SALMAN, H. AL-QASSEM, AND Y. PAN, *Singular integrals on product domains*, Indiana University mathematics journal, **55** (2006), 369–387.
- [10] A. AL-SALMAN AND Y. PAN, *Singular integrals with rough kernels in $L\log^+ L(S^{n-1})$* , J. London Math. Soc. (2) **66** (2002) 153–174.
- [11] A. CARBERY, S. WAINGER AND J. WRIGHT, *Double Hilbert transforms along polynomial surfaces in \mathbf{R}^3* , Duke Math. J. **101** (2000), no. 3, 499–513.
- [12] A. CARBERY, S. WAINGER AND J. WRIGHT, *Triple Hilbert transforms along polynomial surfaces in \mathbf{R}^4* , Rev. Mat. Iberoamericana **25** (2009), no. 2, 471–519.
- [13] H. CARLSSON, P. SJÖGREN, *Estimates for maximal functions along hypersurfaces*, Ark. Mat. **25** (1987) 1–14.
- [14] H. CARLSSON, P. SJÖGREN, J. O. STROMBERG, *Multiparameter maximal functions along dilation invariant hypersurfaces*, Trans. Amer. Math. Soc. **292** (1985) 335–343.
- [15] Y. CHO, S. HONG, J. KIM AND C. YANG, *Multiparameter singular integrals and maximal operators along flat surfaces*, Rev. Mat. Iberoamericana **24** (2008), no. 3, 1047–1073.

- [16] J. DUOANDIKOETXEA AND J. L. RUBIO DE FRANCIA, *Maximal functions and singular integral operators via Fourier transform estimates*, *Invent. Math.* **84** (1986), 541–561.
- [17] J. DUOANDIKOETXEA, *Multiple singular integrals and maximal functions along hypersurfaces*, *Ann. Inst. Fourier (Grenoble)*, **36** (1986) 185–206.
- [18] D. FAN, K. GUO, AND Y. PAN, *Singular integrals with rough kernels on product spaces*, *Hokkaido Math. J.*, **28** (1999), 435–460.
- [19] D. FAN AND Y. PAN, *Singular integral operators with rough kernels supported by subvarieties*, *Amer. J. Math.* **119** (1997), 799–839.
- [20] R. FEFFERMAN, *Singular integrals on product domains*, *Bull. Amer. Math. Soc.*, **4** (1981), 195–201.
- [21] R. FEFFERMAN AND E. STEIN, *Singular integrals on product spaces*, *Adv. in Math.*, **45** (1982), 117–143.
- [22] Y. JIANG AND S. LU, *A class of singular integral operators with rough kernels on product domains*, *Hokkaido Math. J.*, **24** (1995), 1–7.
- [23] H. V. LE, *A note on singular integrals with dominating mixed smoothness in Triebel-Lizorkin spaces*, *Acta Mathematica Scientia* **34** (4) (2014), 1331–1344.
- [24] S. S. LU, Y. PAN AND D. YANG, *Rough Singular Integrals Associated to Surfaces of Revolution*, *Proceedings of the AMS – American Mathematical Society*, vol. **129**, 2001, pp. 2931–2940.
- [25] W. KIM, S. WAINGER, J. WRIGHT AND S. ZIESLER, *Singular integrals and maximal functions associated to surfaces of revolution*, *Bull. London Math. Soc.* **28** (1996), 291–296.
- [26] A. NAGEL, S. WAINGER, *L^2 -boundedness of the Hilbert transforms along surfaces and convolution operators homogeneous with respect to a multiple parameter group*, *Amer. J. Math.* **99** (1977) 761–785.
- [27] S. PATEL, *Double Hilbert Transforms along Polynomial Surfaces in \mathbf{R}^3* , *Glasgow Math. J.* **50** (2008), 395–428.
- [28] F. RICCI AND E. M. STEIN, *Multiparameter singular integrals and maximal functions*, *Ann. Inst. Fourier* **42** (1992) 637–670.
- [29] S. SATO, *Estimates for singular integrals and extrapolation*, *Studia Math.* **192** (2009), 219–233.
- [30] S. YANO, *An extrapolation theorem*, *J. Math. Soc. Japan* **3** (1951), 296–305.