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ON THE SPECTRAL NORMS OF r–CIRCULANT AND GEOMETRIC CIRCULANT MATRICES WITH THE BI–PERIODIC HYPER–HORADAM SEQUENCE

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Abstract. In this paper, we define the bi-periodic hyper-Horadam sequence $\{w_n^{(k)}\}_{n\in\mathbb{N}}$ and present its combinatorial properties. Moreover, we obtain upper and lower bounds for the spectral norms of different forms of the r-circulant and geometric circulant matrices with the bi-periodic hyper-Horadam sequence. Then we give some bounds for the spectral norms of the Kronecker and Hadamard products of these matrices.

Mathematics subject classification (2020): 15A60, 15B05, 15B36, 11B39.
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REFERENCES

[1] M. BAHŞI, I. MEZŐ AND S. SOLAK, *A symmetric algorithm for hyper-Fibonacci and hyper-lucas numbers*, Ann. Math. Inform., **43**, (2014), 19–27.
[2] M. BAHŞI AND S. SOLAK, *On the norms of r-circulant matrices with the hyper-Fibonacci and Lucas numbers*, J. Math. Inequal, **8**, 4 (2014), 693–705.
[3] M. BAHŞI, *On the norms of circulant matrices with the generalized Fibonacci and Lucas numbers*, TWMS J. Pure Appl. Math., **6**, 1 (2015), 84–92.
[4] M. BAHŞI, *On the norms of r-circulant matrices with the hyperharmonic numbers*, J. Math. Inequal., **10**, 2 (2015), 445–458.
[5] N. BELAGGOUN AND H. BELBACHIR, *Bi-periodic hyper-Fibonacci numbers*, Kragujevac J. Math., **49**, 4 (2025), 603–614.
[6] H. BELBACHIR AND A. BELKHIR, *On some generalizations of Horadam numbers*, Filomat, **32**, 14 (2018), 5037–5052.
[7] G. BILGICI, *Two generalizations of lucas sequence*, Appl. Math. Comput., **245**, (2014), 526–538.
[8] M. DAĞLI, E. TAN AND O. ÖLMEZ, *On r-circulant matrices with generalized bi-periodic Fibonacci numbers*, J. Appl. Math. Comput., **68**, 3 (2022), 2003–2014.
[9] P.J. DAVIS, *Circulant matrices, wiley*, National Bureau of Standards, Applied Mathematics Series **55**, New York, Washington, 1979.
[10] M. EDSON AND O. YAYENIE, *A new generalization of Fibonacci sequence and extended binet's formula*, Integers, **9**, (2009), 639–654.
[11] H. W. GOULD, *Combinatorial Identities: A Standardized Set of Tables Listing 500 Binomial Coefficient Summations*, Morgantown, West Virginia, 1972.
[12] A. F. HORADAM, *Basic Properties of a Certain Generalized Sequence of Numbers*, Messenger Math., **3**, (1965), 161–176.
[13] R. A. HORN AND C. R. JOHNSON, *Topics in matrix analysis, 1991*, Cambridge University Presss, Cambridge, **37**, 39 (1991).
[14] H. KARNER, J. SCHNEID AND C. W. UEBERHUBER, *Spectral decomposition of real circulant matrices*, Linear Algebra Appl., **367**, (2003), 301–311.
[15] C. KIZILATEŞ AND N. TUGLU, *On the bounds for the spectral norms of geometric circulant matrices*, J. Inequal. Appl., **2016**, 1 (2016), 1–15.

Paper JMI-17-55

[16] C. KIZILATEŞ AND N. TUGLU, *On the Norms of Geometric and Symmetric Geometric Circulant Matrices with the Tribonacci Number*, Gazi Univ. J. Sci., **31**, 2 (2018), 555–567.
[17] E. G. KOCER, T. MANSOUR AND N. TUĞLU, *Norms of circulant and semicirculant matrices with Horadams's numbers*, Ars Comb., **85**, 1 (2018), 353–359.
[18] C. KÖMEE AND Y. YAZLIK, *On the spectral norms of r-circulant matrices with the biperiodic Fibonacci and Lucas numbers*, J. Inequal. Appl., **2017**, 1 (2017), 1–12.
[19] R. MATHIAS, *The spectral norm of a nonnegative matrix*, Linear Algebra Appl., **139**, (1990), 269–284.
[20] E. POLATLI, *On geometric circulant matrices whose entries are bi-periodic Fibonacci and bi-periodic Lucas numbers*, Univers. J. Math. Appl., **3**, 3 (2020), 102–108.
[21] R. REAMS, *Hadamard inverses, square roots and products of almost semidefinite matrices*, Linear Algebra Appl., **288**, (1999), 35–43.
[22] S. UYGUN, *On the bounds for the spectral norms of geometric and r-circulant matrices with bi-periodic Jacobsthal numbers*, J. Appl. Math. Inform., **38**, 1_2 (2020), 99–112.
[23] O. YAYENIE, *A note on generalized Fibonacci sequence*, Appl. Math. Comput., **217**, (2011), 5603–5611.
[24] Y. YAZLIK AND N. TASKARA, *A note on generalized k-Horadam sequence*, Comput. Math. Appl., **63**, 1 (2012), 36–41.
[25] Y. YAZLIK AND N. TASKARA, *On the norms of an r-circulant matrix with the generalized k-Horadam numbers*, J. Inequal. Appl., **2013**, 1 (2013), 1–8.
[26] G. ZIELKE, *Some remarks on matrix norms, condition numbers, and error estimates for linear equations*, Linear Algebra Appl., **110**, (1988), 29–41.