

EXACT EXPONENTS FOR INCLUSION OF DISCRETE MUCKENHOUPHT CLASSES INTO GEHRING CLASSES AND REVERSE

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Abstract. In this paper, we establish some embedding relationships between Muckenhoupt and Gehring classes \mathcal{A}_p and \mathcal{G}_r by proving transition and inclusion relations. We also identify the exact range of $\tau > 1$ for which $w^\tau \in \mathcal{A}_p$. Additionally, we show that the weights that satisfy the \mathcal{A}_p -condition also meet the \mathcal{A}_∞ -condition. Next, we prove the Jones factorization property of \mathcal{A}_p weights in terms of two \mathcal{A}_1 weights by employing the discrete Rubio De Francia iterated algorithm. Finally, we determine the specific ranges of indices (sharp exponents) for which w belongs to $\mathcal{G}_r(\mathcal{A}_p)$ if w belongs to $\mathcal{A}_p(\mathcal{G}_r)$ and the precise range of $q < p$ for which w belongs to \mathcal{A}_q given that it belongs to \mathcal{A}_p .

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