

## ON THE CONVEXITY OF SOME TRACE FUNCTIONS

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*Abstract.* In this paper, we study the convexity of the trace function  $A \rightarrow \text{Tr } B^* f(A) B$ . And we get an extension of the Peierls-Bogolyubov inequality and the joint convexity of the trace geometric mean.

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