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ON p–QUASI–n–HYPONORMAL OPERATORS

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Abstract. An operator $T \in B(H)$ is called p-quasi-n-hyponormal if

$$T^*(T^{*n}T^n)^pT \geqslant T^*(T^nT^{*n})^pT$$

for a positive number $0 < p \leqslant 1$ and a positive integer n, which is a further generalization of normal operator. In this paper we introduce the class of p-quasi-n-hyponormal operators and show its structural properties via Hansen inequality and Löwner-Heinz inequality. As important applications, we obtain that every p-quasi-n-hyponormal operator has a scalar extension. In addition, we prove that if T is a quasiaffine transform of p-quasi-n-hyponormal, then T satisfies Weyl's theorem. Finally some examples are presented.

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