

## ON SEVERAL INEQUALITIES RELATED TO CONVEX FUNCTIONS

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**Abstract.** In this paper, for a function  $f : \mathcal{X} \rightarrow \mathbb{R}$ , we introduce the following expression:  $\Delta_\lambda(f)(x, y) := \lambda f(x) + (1 - \lambda)f(y) - f(\lambda x + (1 - \lambda)y)$ , where  $x, y \in \mathcal{X}$  and  $\lambda \in \mathbb{R}$ . The purpose of this article is to characterize this expression, by finding various estimates of it. We also give some characterizations of  $\Delta_\lambda(f)(x, y)$  when function  $f$  is convex, which prove refinements of Young's inequality. Finally, we give several inequalities in a normed space.

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