

SHARP BOUNDS ON THE HANKEL DETERMINANT OF THE INVERSE FUNCTIONS FOR CERTAIN ANALYTIC FUNCTIONS

LEI SHI, MUHAMMAD ARIF*, H. M. SRIVASTAVA AND MUHAMMAD IHSAN

Abstract. In most cases, the problem of finding bounds for the inverse function is much more difficult than finding bounds for the function itself. Thus, there are relatively little sharp bounds of Hankel determinant on the inverse functions. In the present paper, we introduce a subclass of bounded turning function \mathcal{B}_{car} associated with a cardioid domain. The purpose of this article is to investigate certain coefficient related problems on the inverse functions for $f \in \mathcal{B}_{car}$. The bounds of some initial coefficients, the Fekete-Szegő type inequality and the estimation of Hankel determinants of second and third order are obtained. All of these bounds are proved to be sharp.

Mathematics subject classification (2020): 30C45, 30C50, 30C80.

Keywords and phrases: Univalent functions, inverse function, coefficient bounds, Hankel determinant.

REFERENCES

- [1] O. P. AHUJA, K. KHATTER, V. RAVICHANDRAN, *Toeplitz determinants associated with Ma-Minda classes of starlike and convex functions*, Iranian Journal of Science and Technology, Transactions A: Science. **45**, 6 (2021), 1–11.
- [2] R. M. ALI, *Coefficients of the inverse of strongly starlike functions*, Bulletin of the Malaysian Mathematical Sciences Society. **26**, 1 (2003), 63–71.
- [3] Ş. ALTINKAYA, S. YALÇIN, *Upper bound of second Hankel determinant for bi-Bazilevic functions*, Mediterranean Journal of Mathematics. **13**, 6 (2016), 4081–4090.
- [4] K. O. BABALOLA, *On $H_3(1)$ Hankel determinant for some classes of univalent functions*, Inequality Theory and Applications. **6**, 1 (2010), 1–7.
- [5] L. BIEBERBACH, *Über die koeffizienten derjenigen Potenzreihen, welche eine schlichte Abbildung des Einheitskreises vermitteln*, Sitzungsberichte Preussische Akademie der Wissenschaften. **138**, 1 (1916), 940–955.
- [6] N. E. CHO, B. KOWALCZYK, O. S. KWON, A. LECKO, Y. J. SIM, *The bounds of some determinants for starlike functions of order alpha*, Bulletin of the Malaysian Mathematical Sciences Society. **41**, 1 (2018), 523–535.
- [7] L. DE BRANGES, *A proof of the Bieberbach conjecture*, Acta Mathematica. **154**, 1 (1985), 137–152.
- [8] A. EBADIAN, T. BULBOACĂ, N. E. CHO, E. A. ADEGANI, *Coefficient bounds and differential subordinations for analytic functions associated with starlike functions*, Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Series A. Matemáticas. **114**, 1 (2020), 1–10.
- [9] K. GANGANIA, S. S. KUMAR, *Bohr-Rogosinski Phenomenon for $\mathcal{S}^*(\psi)$ and $\mathcal{C}(\psi)$* , Mediterranean Journal of Mathematics. **19**, 4 (2022), 1–18.
- [10] A. W. GOODMAN, *Univalent Functions*, Mariner, Tampa, Florida, 1983.
- [11] W. K. HAYMAN, *On second Hankel determinant of mean univalent functions*, Proceedings of the London Mathematical Society. **3**, 1 (1968), 77–94.
- [12] O. P. JUNEJA AND S. RAJASEKARAN, *Coefficient estimates for inverses of α -spiral functions*, Complex Variables and Elliptic Equations. **6**, 2 (1986), 99–108.
- [13] G. P. KAPOOR AND A. K. MISHRA, *Coefficient estimates for inverses of starlike functions of positive order*, Journal of Mathematical Analysis and Applications. **329**, 2 (2007), 922–934.

- [14] F. KEOUGH, E. MERKES, *A coefficient inequality for certain subclasses of analytic functions*, Proceedings of the American Mathematical Society. **20**, 1 (1969), 8–12.
- [15] D. V. KRISHNA, T. R. REDDY, *Coefficient inequality for certain subclasses of analytic functions associated with Hankel determinant*, Indian Journal of Pure and Applied Mathematics. **46**, 1 (2015), 91–106.
- [16] J. G. KRZYŻ, R. J. LIBERA, E. ZLOTKIEWICZ, *Coefficients of inverse of regular starlike functions*, Ann. Univ. Mariae. Curie-Skłodowska. **33**, 10 (1979), 103–109.
- [17] B. KOWALCZYK, A. LECKO, Y. J. SIM, *The sharp bound of the Hankel determinant of the third kind for convex functions*, Bulletin of the Australian Mathematical Society. **97**, 1 (2018), 435–445.
- [18] B. KOWALCZYK, A. LECKO, D. K. THOMAS, *The sharp bound of the third Hankel determinant for starlike functions*, Forum Math. **34**, 5 (2022), 1249–1254.
- [19] O. S. KWON, A. LECKO, Y. J. SIM, *The bound of the Hankel determinant of the third kind for starlike functions*, Bulletin of the Malaysian Mathematical Sciences Society. **41**, 2 (2019), 767–780.
- [20] O. S. KWON, A. LECKO, Y. J. SIM, *On the fourth coefficient of functions in the Carathéodory class*, Computational Methods and Function Theory. **18**, 2 (2018), 307–314.
- [21] A. LECKO, Y. J. SIM, B. ŚMIAROWSKA, *The sharp bound of the Hankel determinant of the third kind for starlike functions of order $1/2$* , Complex Analysis and Operator Theory. **13**, 5 (2019), 2231–2238.
- [22] S. MADHUMITHA, V. RAVICHANDRAN, *Radius of starlikeness of certain analytic functions*, Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas. **115**, 4 (2021), 1–18.
- [23] S. K. LEE, V. RAVICHANDRAN, S. SUPRAMANIAM, *Bounds for the second Hankel determinant of certain univalent functions*, Journal of Inequalities and Applications. **281**, 1 (2013), 1–17.
- [24] K. LÖWNER, *Untersuchungen über schlichte konforme Abbildungen des Einheitskreises*. I. Mathematische Annalen. **89**, 1 (1923), 103–121.
- [25] M. OBRADOVIĆ AND N. TUNESKI, *Hankel determinants of second and third order for the class \mathcal{S} of univalent functions*, Mathematica Slovaca. **71**, 3 (2021), 649–654.
- [26] CH. POMMERENKE, *On the coefficients and Hankel determinants of univalent functions*, Journal of the London Mathematical Society. **1**, 1 (1966), 111–122.
- [27] CH. POMMERENKE, *On the Hankel determinants of univalent functions*, Mathematika. **14**, 1 (1967), 108–112.
- [28] S. PONNUSAMY, N. L. SHARMA, K. J. WIRTHS, *Logarithmic coefficients of the inverse of univalent functions*, Results in Mathematics. **73**, 4 (2018), 1–20.
- [29] A. RIAZ, M. RAZA, D. K. THOMAS, *Hankel determinants for starlike and convex functions associated with sigmoid functions*, Forum Mathematicum. **34**, 1 (2022), 137–156.
- [30] K. SHARMA, N. K. JAIN, V. RAVICHANDRAN, *Starlike functions associated with a cardioid*, Afrika Matematika. **27**, 5 (2016), 923–939.
- [31] L. SHI, M. ARIF, J. IQBAL, K. ULLAH, M. S. GHUFRAN, *Sharp Bounds of Hankel Determinant on Logarithmic Coefficients for Functions Starlike with Exponential Function*, Fractal and Fractional. **6**, 11 (2022), 1–16.
- [32] L. SHI, H. M. SRIVASTAVA, A. RAFIQ, M. ARIF, M. IHSAN, *Results on Hankel determinants for the inverse of certain analytic functions subordinated to the exponential function*, Mathematics. **10**, 19 (2022), 1–15.
- [33] H. SILVERMAN, *Coefficient bounds for inverses of classes of starlike functions*, Complex Variables and Elliptic Equations. **12**, 1 (1989), 23–31.
- [34] Y. J. SIM, D. K. THOMAS, P. ZAPRAWA, *The second Hankel determinant for starlike and convex functions of order α* , Complex Variables and Elliptic Equations. **67**, 10 (2022), 2423–2443.
- [35] H. M. SRIVASTAVA, G. KAUR AND G. SINGH, *Estimates of the fourth Hankel determinant for a class of analytic functions with bounded turnings involving cardioid domains*, J. Nonlinear Convex Anal. **22** (2021), 511–526.
- [36] Z. G. WANG, M. RAZA, M. ARIF, K. AHMAD, *On the third and fourth Hankel determinants of a subclass of analytic functions*, Bull. Malays. Math. Sci. Soc. **45**, 1 (2022), 323–359.
- [37] P. ZAPRAWA, *Third Hankel determinants for subclasses of univalent functions*, Mediterranean Journal of Mathematics. **14**, 1 (2017), 1–10.
- [38] P. ZAPRAWA, M. OBRADOVIĆ, N. TUNESKI, *Third Hankel determinant for univalent starlike functions*, Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas. **115**, 2 (2021), 1–6.

Journal of Mathematical Inequalities

www.ele-math.com

jmi@ele-math.com