

PARAMETERIZED MORE ACCURATE HARDY–HILBERT–TYPE INEQUALITIES AND APPLICATIONS

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Abstract. By means of the weight coefficients, the idea of introduced parameters and Hermite-Hadamard's inequality, a more accurate Hardy-Hilbert-type inequality with the general homogeneous kernel and the discrete intermediate variables is given. The equivalent form and a few equivalent statements of the best possible constant factor related to some parameters are obtained. As applications, the operator expressions, a few particular cases and some examples are considered.

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