A NOTE ON r–CIRCULANT MATRICES
INVOLVING GENERALIZED NARAYANA NUMBERS

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Abstract. In order to further connect structured matrices and integer sequences, r-circulant matrices involving the generalized Narayana numbers are considered. Estimates for spectral norms bounds of such matrices are presented and their eigenvalues are determined. Moreover, the conditions under which the circulant matrix and the skew circulant matrix involving generalized Narayana numbers are invertible are given. In particular, it is shown that every circulant matrix with Narayana numbers is necessarily invertible.

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