

L^p BOUNDEDNESS FOR MAXIMAL SINGULAR INTEGRALS WITH MIXED HOMOGENEITY ALONG COMPOUNDS CURVES

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Abstract. In this paper, we study the maximal truncated singular integral operators with rough kernel along certain compound curves, which contain many classical model examples. We prove the L^p boundedness of such maximal singular integral operators under very weak conditions on the integral kernels both on the unit sphere and the radial direction. The main results essentially improve and extend certain previous results.

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