

GENERALIZED NUMERICAL RADIUS INEQUALITIES INVOLVING POSITIVE SEMIDEFINITE BLOCK MATRICES

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Abstract. In this paper, we are interested in generalized numerical radius inequalities for the off-diagonal part of a positive semidefinite block matrix. These inequalities produce a new set of inequalities for products and sums of matrices and some inequalities related to sectorial matrices.

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