

A CLASS OF HALF-DISCRETE HILBERT-TYPE INEQUALITIES IN THE WHOLE PLANE INVOLVING SOME CLASSICAL KERNELS

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Abstract. In this work, we first construct a half-discrete kernel function, which is defined in the whole plane and involves both the homogeneous and the non-homogeneous cases. By employing the method of weight coefficient and some classical techniques of real analysis, a class of half-discrete Hilbert-type inequalities with the newly constructed kernel as well as the equivalent inequalities of Hardy's type are established. In addition, we prove that all the constant factors in the newly established inequalities are the best possible. Lastly, assigning special values to the parameters, and using the partial fraction expansions of cotangent function and cosecant function, some new half-discrete Hilbert-type inequalities with special kernels defined in the whole plane are presented at the end of the paper.

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