SHARP INEQUALITIES FOR THE
ATOM–BOND (SUM) CONNECTIVITY INDEX

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Abstract. For a graph G, its atom-bond connectivity (ABC) index (respectively, atom-bond sum connectivity (ABS) index) is defined as the addition of the numbers $\sqrt{d_i + d_j - 2}(d_i d_j)^{-1/2}$ (respectively, $\sqrt{d_i + d_j - 2}(d_i + d_j)^{-1/2}$) over all unordered pairs of adjacent vertices $\{v_i, v_j\}$ of G, where d_i and d_j denote the degrees of v_i and v_j, respectively. In this paper, sharp upper bounds on the ABC and ABS indices are derived. All the graphs that attain the obtained bounds are also completely characterized.

Mathematics subject classification (2020): 05C07, 05C09, 05C92.

Keywords and phrases: Topological index, atom-bond connectivity index, atom-bond sum connectivity index, bound.

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