

## FIXED POINT THEOREMS FOR $\alpha - \beta_M$ -GERAGHTY TYPE CONTRACTIONS WITH APPLICATIONS IN MATRIX EQUATIONS

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*Abstract.* In this manuscript, we present a novel notion of  $\alpha - \beta_M$ -Geraghty type contractions, which are employed to establish the existence and uniqueness of a fixed point in complete  $b$ -metric spaces. To emphasize the significance of our results, illustrative examples are provided. Furthermore, we utilize the obtained results to demonstrate the existence of a solution for matrix equations. Thus, our results provide a suitable extension in this respect.

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