GENERALIZATION OF TWO–POINT OSTROWSKI'S INEQUALITY

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Abstract. The paper presents a novel approach to generalize the two-point weighted Ostrowski's formula for Riemann-Stieltjes integrals by utilizing a unique class of functions of bounded r-variation. The proposed approach yields several results that exhibit sharp and better bounds compared to existing established results by using parameters and weights. Additionally, the paper also captures many of the known results as special cases.

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