

VARIABLE ANISOTROPIC HERZ–MORREY–HARDY  
SPACES AND THEIR APPLICATIONS

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**Abstract.** Let  $A$  be an expansive dilation on  $\mathbb{R}^n$  and let  $\alpha(\cdot) \in L^\infty(\mathbb{R}^n)$ . Also let  $p(\cdot) : \mathbb{R}^n \rightarrow (0, \infty)$  be a variable exponent function satisfying the globally log-Hölder continuous condition. In this paper, the authors first introduce the variable anisotropic Herz-Morrey-Hardy spaces  $HMK_{p(\cdot), \lambda}^{\alpha(\cdot), q}(A; \mathbb{R}^n)$  and  $HMK_{p(\cdot), \lambda}^{\alpha(\cdot), q}(A; \mathbb{R}^n)$ , via the non-tangential grand maximal function, and then establish their atomic decompositions. As applications, the authors obtain the boundedness of some sublinear operators from  $HMK_{p(\cdot), \lambda}^{\alpha(\cdot), q}(A; \mathbb{R}^n)$  to  $M\dot{K}_{p(\cdot), \lambda}^{\alpha(\cdot), q}(A; \mathbb{R}^n)$  and from  $HMK_{p(\cdot), \lambda}^{\alpha(\cdot), q}(A; \mathbb{R}^n)$  to  $MK_{p(\cdot), \lambda}^{\alpha(\cdot), q}(A; \mathbb{R}^n)$ .

**Mathematics subject classification (2020):** Primary 42B30; Secondary 42B35, 46E30.

**Keywords and phrases:** Anisotropic, Herz space, Hardy space, variable exponent, atom.

## REFERENCES

- [1] A. ALMEDIDA AND D. DRIHEM, *Maximal, potential and singular type operators on Herz spaces with variable exponents*, J. Math. Anal. Appl., **394** (2012), 781–795.
- [2] E. ACERBI AND G. MINGIONE, *Regularity results for stationary electro-rheological fluids*, Arch. Ration. Mech. Anal., **164** (2002), 213–259.
- [3] M. BOWNIK, *Anisotropic Hardy spaces and wavelets*, Mem. Amer. Math. Soc., **164** (2003), vi+122 pp.
- [4] Y. CHEN, S. LEVINE AND M. RAO, *Variable exponent, linear growth functionals in image restoration*, SIAM J. Appl. Math., **66** (2006), 1383–1406.
- [5] R. R. COIFMAN, P.-L. LIONS, Y. MEYER AND S. SEMMES, *Compensated compactness and Hardy spaces*, J. Math. Pures Appl., (9) **72** (1993), 247–286.
- [6] D. V. CRUZ-URIBE AND A. FIORENZA, *Variable Lebesgue Spaces, Foundations and Harmonic Analysis*, Applied and Numerical Harmonic Analysis, Birkhäuser/Springer, Heidelberg, 2013. x+312 pp.
- [7] L. DIENING, P. HARJULEHTO, P. HÄSTÖ AND M. RUŽIČKA, *Lebesgue and Sobolev Spaces with Variable Exponents*, Lecture Notes in Mathematics, 2017, Springer, Heidelberg, 2011. x+509 pp.
- [8] Y. DING, S. LAN AND S. LU, *New Hardy spaces associated with some anisotropic Herz spaces and their applications*, Acta Math. Sin., **24** (2008), 1449–1470.
- [9] B. DONG AND J. XU, *Variable exponent Herz-type Hardy spaces with and their applications*, Anal. Theory Appl., **31** (2015), 321–353.
- [10] C. FEFFERMAN AND E. M. STEIN,  *$H^p$  spaces of several variables*, Acta Math, **129** (1972), 137–193.
- [11] M. IZUKI, *Boundedness of sublinear operators on Herz spaces with variable exponent and application to wavelet characterization*, Anal. Math., **36** (2010), 33–50.
- [12] O. KOVÁČIK AND J. RÁKOSNÍK, *On spaces  $L^{p(x)}$  and  $W^{k, p(x)}$* , Czechoslovak Math. J., **41** (1991), 592–618.
- [13] Y. LU AND Y. ZHU, *Boundedness of some sublinear operators and commutators on Morrey-Herz spaces with variable exponents*, Czech. Math. J. **64** (1991), 592–618.
- [14] E. NAKAI AND Y. SAWANO, *Hardy spaces with variable exponents and generalized Campanato spaces*, J. Funct. Anal., **262** (2012), 3665–3748.
- [15] Y. SAWANO, *Atomic decompositions of Hardy space with variable exponent and its application to bounded linear operators*, Integral Equations Operator Theory, **77** (2013), 123–148.

- [16] E. M. STEIN, *Harmonic Analysis: Real-Variable Methods, Orthogonality, and Oscillatory Integrals*, Princeton Univ. Press, Princeton, N. J., 1993.
- [17] E. M. STEIN AND G. WEISS, *On the theory of harmonic functions of several variables. I. The theory of  $H^p$ -spaces*, Acta Math., **103** (1960), 25–62.
- [18] J. TAN, *Atomic decompositions of localized Hardy spaces with variable exponents and applications*, J. Geom. Anal., **29** (2019), 799–827.
- [19] H. WANG AND Z. LIU, *The Herz-type Hardy spaces with variable exponent and their applications*, Taiwanese J. Math., **16** (2012), 1363–1389.
- [20] H. WANG, *Anisotropic Herz spaces with variable exponents*, Commun. Math. Anal., **18** (2015), 1–14.
- [21] J. XU AND X. YANG, *Herz-Morrey-Hardy spaces with variable exponents and their applications*, J. Function Spaces, **2015** (2015), 1–19.
- [22] J. XU AND X. YANG, *The molecular decomposition of Herz-Morrey-Hardy spaces with variable exponents and its application*, J. Math. Inequal., **10** (2016), 977–1008.