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LOWER BOUNDS FOR THE BLOW–UP TIME IN A HIGHER–ORDER NONLINEAR KIRCHHOFF–TYPE EQUATION

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Abstract. This paper is concerned with a nonlinear higher-order Kirchhoff-type equation with dissipation in a bounded domain. By establishing a first order differential inequality technique, a lower bound for the blow-up time is obtained when the blow-up of solution occurs.

Mathematics subject classification (2020): 35L05, 35L25, 35L75.
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REFERENCES

[1] R. A. ADAMS AND J. J. F. FOURNIER, *Sobolev Spaces*, Academic Press, 2003.
[2] G. AUTUORI AND P. PUCCI, *Local asymptotic stability for polyharmonic Kirchhoff systems*, Appl. Anal., Special Volume dedicated to Prof. P. L. Butzer, **90** (2011), 493–514.
[3] G. AUTUORI, F. COLASUONNO AND P. PUCCI, *Lifespan estimates for solutions of polyharmonic Kirchhoff systems*, Math. Models Methods Appl. Sci., **22** (2012), 1–36.
[4] G. AUTUORI, P. PUCCI AND M. C. SALVATORI, *Asymptotic stability for nonlinear Kirchhoff systems*, Nonlinear Anal., RWA., **10** (2009), 889–909.
[5] G. AUTUORI, P. PUCCI AND M. C. SALVATORI, *Global Nonexistence for Nonlinear Kirchhoff Systems*, Archive Rat. Mech. Anal., **196** (2010), 489–516.
[6] V. A. GALAKTIONOV AND S. I. POHOZAEV, *Blow-up and critical exponents for nonlinear hyperbolic equations*, Nonlinear Anal., **53** (2003), 453–466.
[7] Q. GAO, F. LI AND Y. WANG, *Blow up of the solution for higher-order Kirchhoff-type equations with nonlinear dissipation*, Cent. Eur. J. Math., **9** (2011), 686–698.
[8] G. C. GORAIN, *Exponential energy decay estimates for the solutions of n-dimensional Kirchhoff type wave equation*, Appl. Math. Comput., **177** (2006), 235–242.
[9] E. HESAMEDDINI, *Blow-up of the Solution for Higher-Order Integro-Differential Equation with Nonlinear Dissipation*, Applied Mathematical Sciences, **5** (2011), 3575–3583.
[10] V. KOMORNIK, *Exact Controllability and Stabilization*, The Multiplier Method, Masson, Paris, 1994.
[11] F. C. LI, *Global existence and blow-up of solutions for a higher-order kirchhoff-type equation with nonlinear dissipation*, Appl. Math. Letters, **17** (2004), 1409–1414.
[12] T. MATSUYAMA AND R. IKEHATA, *On global solutions and energy decay for the wave equations of Kirchhoff type with nonlinear damping terms*, J. Math. Anal. Appl., **204** (1996), 729–753.
[13] S. A. MESSAOUDI AND B. SAID HOUARI, *A blow-up result for a higher-order nonlinear Kirchhoff-type hyperbolic equation*, Appl. Math. Lett., **20** (2007), 866–871.
[14] N. NAKAO, *A difference inequality and its application to nonlinear evolution equations*, J. Math. Soc. Japan, **30** (1978), 747–762.
[15] K. ONO, *On global existence, asymptotic stability and blowing up of solutions for some degenerate nonlinear wave equations of Kirchhoff type*, Math. Meth. Appl. Sci., **20** (1997), 151–177.
[16] K. ONO, *On global existence, decay and blow-up of solutions for some mildly degenerate Kirchhoff strings*, J. Differential Equations, **137** (1997), 273–301.
[17] K. ONO, *On global solutions and blow-up solutions of nonlinear Kirchhoff strings with nonlinear dissipation*, J. Math. Anal. Appl., **216** (1997), 321–342.

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[18] V. PATA AND S. ZELIK, *Smooth attractors for strongly damped wave equations*, Nonlinearity, **19** (2006), 1495–1506.
[19] E. PISKIN, *Blow up of Solutions for a System of Nonlinear Higher-order Kirchhoff-type Equations*, Mathematics and Statistics, **6** (2014), 219–229.
[20] E. PISKIN, *Sobolev Spaces*, Sekin Publishing, 2017.
[21] E. VITILLARO, *Global nonexistence theorems for a class of evolution equations with dissipation*, Arch. Ration. Mech. Anal., **149** (1999), 155–182.
[22] S. T. WU AND L. Y. TSAI, *Blow-up of solutions for some nonlinear wave equations of Kirchhoff type with some dissipation*, Nonlinear Anal., TMA., **65** (2006), 243–264.
[23] Y. J. YE, *Global existence and energy decay estimate of solutions for a higher-order Kirchhoff type equation with damping and source term*, Nonlinear Analysis: RWA **14** (2013) 2059–2067.
[24] Y. J. YE AND X. X. TAO, *Initial Boundary Value Problem for Higher-order Nonlinear Kirchhoff-type Equation*, ACTA Mathematica Sinica (Chinese Series), **62** (2019), 923–938.
[25] H. YUKSEKKAYA, E. PISKIN, S. M. BOULAARAS AND B. B. CHERIF, *Existence, Decay, and Blow-Up of Solutions for a Higher-Order Kirchhoff-Type Equation with Delay Term*, Journal of Function Spaces, **2021** (2021), 11 pages, 866–871, https://doi.org/10.1155/2021/4414545.
[26] R. ZENG, C. L. MU AND S. M. ZHOU, *A blow-up result for Kirchhoff-type equations with high energy*, Math. Methods Appl. Sci., **34** (2011), 479–486.