NON–LINEAR CASAZZA–KALTON–CHRISTENSEN–VAN EIJNDHOVEN PERTURBATION WITH APPLICATIONS

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Abstract. Let $\mathcal{X}$, $\mathcal{Y}$ be Banach spaces and $S: \mathcal{X} \to \mathcal{Y}$ be an invertible Lipschitz map. Let $T: \mathcal{X} \to \mathcal{Y}$ be a map and there exist $\lambda_1, \lambda_2 \in [0,1)$ such that

$$
\|Tx - Ty - (Sx - Sy)\| \leq \lambda_1 \|Sx - Sy\| + \lambda_2 \|Tx - Ty\|, \quad \forall x, y \in \mathcal{X}.
$$

Then we prove that $T$ is an invertible Lipschitz map. This is non-linear version of 26 years old Casazza-Kalton-Christensen-van Eijndhoven perturbation. It also a non-linear version of 29 years old Soderlind-Campanato perturbation and 3 years old Barbagallo-Ernst-Thera perturbation. We give applications to the theory of metric frames. The notion of Lipschitz atomic decomposition for Banach spaces is also introduced.


Keywords and phrases: Paley-Wiener perturbation, Lipschitz map, metric frame, atomic decomposition.

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